An Evolutionary Model of Price Competition between Spatially Distributed Firms

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How does cooperative behavior evolve?

- Important question in biology, economics, and social science in general
- Frequently studied in the context of prisoner's dilemmas
- The most common explanation of cooperative behavior relies on the idea of reciprocity
- An alternative explanation has been proposed by the biologists Martin Nowak and Robert May (*Nature*, 1992). Their explanation relies on the idea of local interaction between spatially distributed agents
- Some economists have adopted this idea in order to explain cooperative behavior in economic contexts (e.g., Bergstrom & Stark, *Amer. Econ. Rev.*, 1993; Eshel et al, *Amer. Econ. Rev.*, 1998)

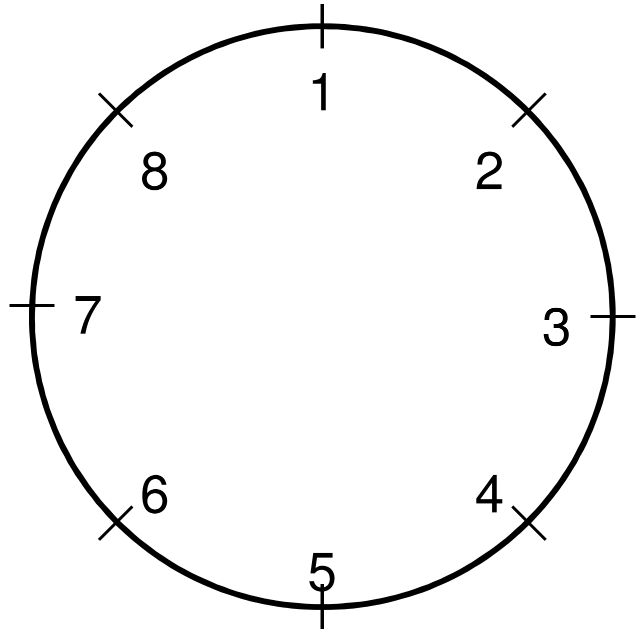
Research question

- We study price competition between spatially distributed firms
- Neighboring firms can cooperate by jointly increasing their price (collusion)
- Our research question is as follows:

Can an evolutionary model with local interaction result in cooperative behavior among spatially distributed price competing firms?

Model (1)

- There are n firms, which are located on the circumference of a circle
- The circumference has length n, and the firms are equally spaced
- There is a continuum of consumers, which are uniformly distributed on the circumference of the circle
- Firms all produce the same good, they all have the same constant marginal cost c, and they all have an unlimited production capacity
- Each consumer requires exactly one unit of the good produced by the firms

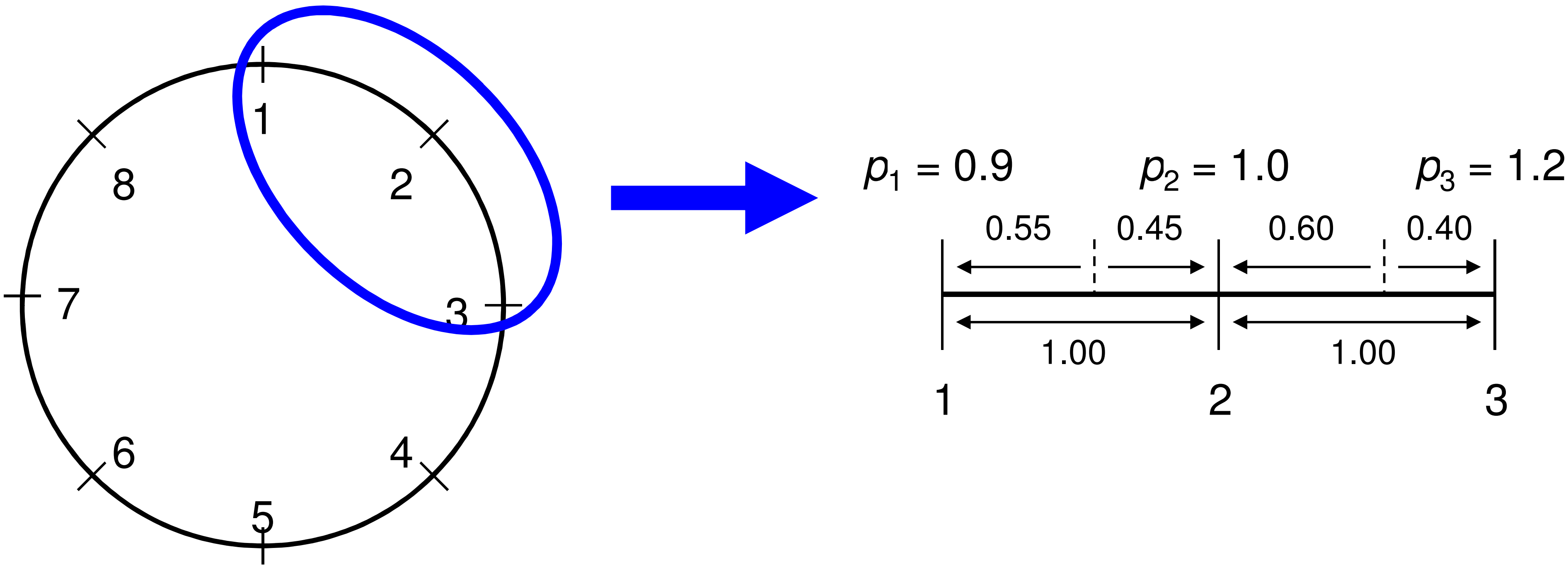


$n = 8$

Model (2)

- Each firm chooses the price per unit that consumers have to pay
- Firms choose their prices simultaneously and independently from a set $\{0, \delta, 2\delta, \ldots\}$
- A consumer's total cost of buying a unit from a firm equals the price charged by the firm plus transportation cost
- Transportation cost equals *t* times the distance, over the circumference of the circle, between the consumer and the firm
- Each consumer buys one unit from the firm for which total cost is lowest

Model (3)



- $p_1 = 0.9$
- $p_2 = 1.0$
- $p_3 = 1.2$
- $t = 1$
- $\pi_2 = (1.0 - c)(0.45 + 0.60)$

Nash equilibrium

- The model has a Nash equilibrium in which each firm charges a price of $c + t$

Evolutionary dynamics

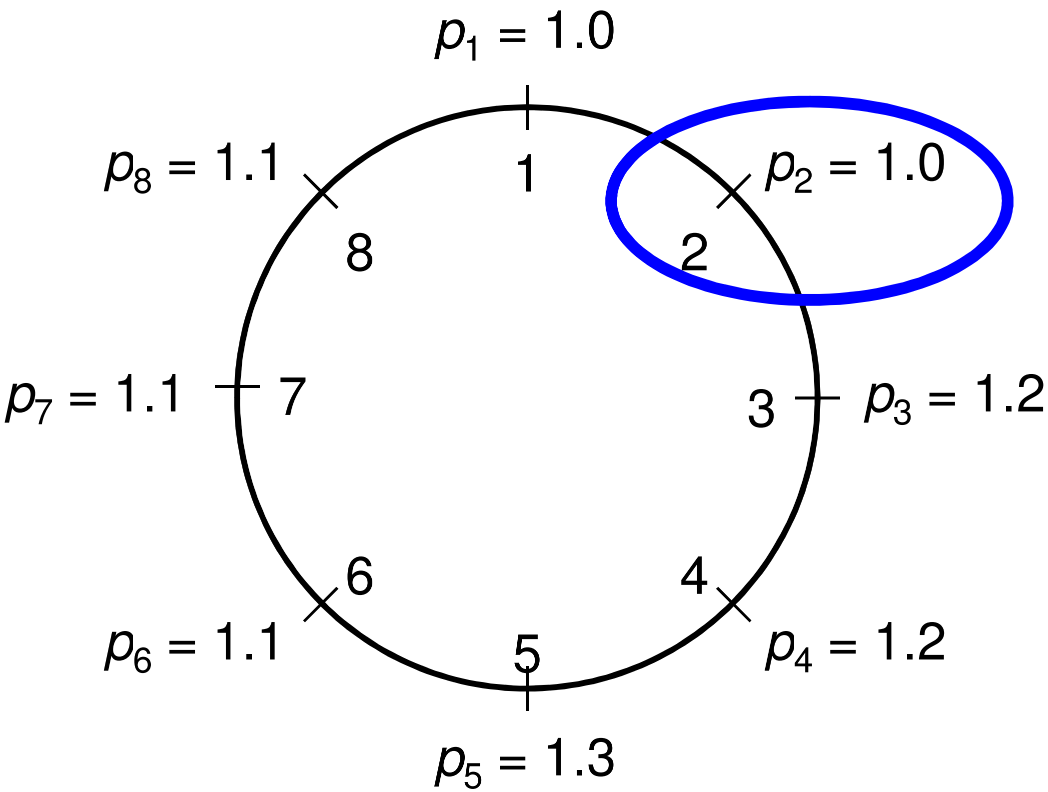
- The price competition game is repeated an infinite number of times
- Firms are boundedly rational
- After each period of time, firms can change their price according to the following two mechanisms:
  - Imitation
  - Experimentation (mutation)

Evolutionary dynamics: Imitation (1)

- One firm is selected at random to choose a new price
- The firm knows its own price in the previous period and the prices of its two neighbors, and it chooses one of these three prices as its new price
- The firm knows its own profit in the previous period and the observed profits of its two neighbors
- The observed profit of a neighbor equals the true profit of the neighbor plus a normally distributed random variable with mean zero and standard deviation σ
- The (observed) profits of firms that charged the same price in the previous period are averaged
- The price for which the average (observed) profit is highest is chosen

Evolutionary dynamics: Imitation (2)

- $c = 0$; $t = 1$
- Suppose firm 2 is randomly selected
- $p_1 = 1.0$; $\pi_1 = 1.05$
- $p_2 = 1.0$; $\pi_2 = 1.10$
- $p_3 = 1.2$; $\pi_3 = 1.08$
- Firm 2 chooses 1.2 as its new price if



$$(\pi_1 + N(0, \sigma) + \pi_2) / 2 < \pi_3 + N(0, \sigma)$$

and stays at its old price of 1.0 otherwise

Evolutionary dynamics: Experimentation

- Each firm experiments with a common, independent probability μ
- A firm that experiments randomly draws its new price from a uniform distribution over the set

  $\{p - m\delta, \ldots, p - \delta, p + \delta, \ldots, p + m\delta\}$,

  where p is the firm's old price and m is a positive integer
- If the new price is negative, the firm stays at its old price

Summary of parameters

n	Number of firms
c	Firms’ constant marginal cost
t	Consumers’ transportation cost per distance unit
δ	Difference between consecutive price levels
σ	Noise level in observed profits
μ	Experimentation probability
m	Maximum price increase/decrease due to experimentation

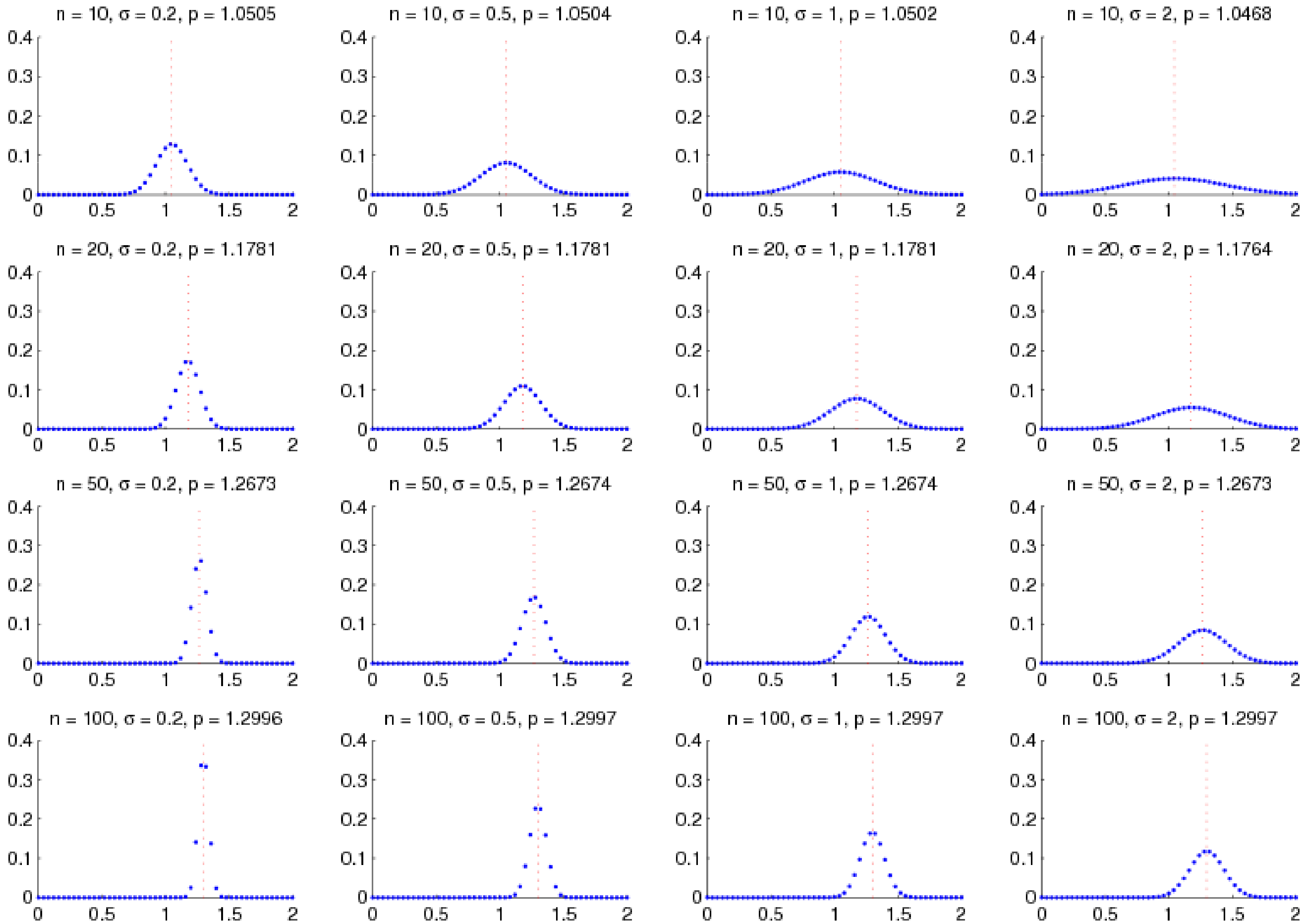
Analysis

- Without loss of generality, we choose $c = 0$ and $t = 1$
- Consequently, in the Nash equilibrium each firm charges a price of 1.00
- We choose $\delta = 0.04$, so that firms' prices are in the set {0.00, 0.04, 0.08, …}
- We choose $m = 1$, so that experimentation results in a new price that is either 0.04 above or 0.04 below the old price
- We analyze our model for various values of n, σ, and μ
- Our focus is on what happens in the long run

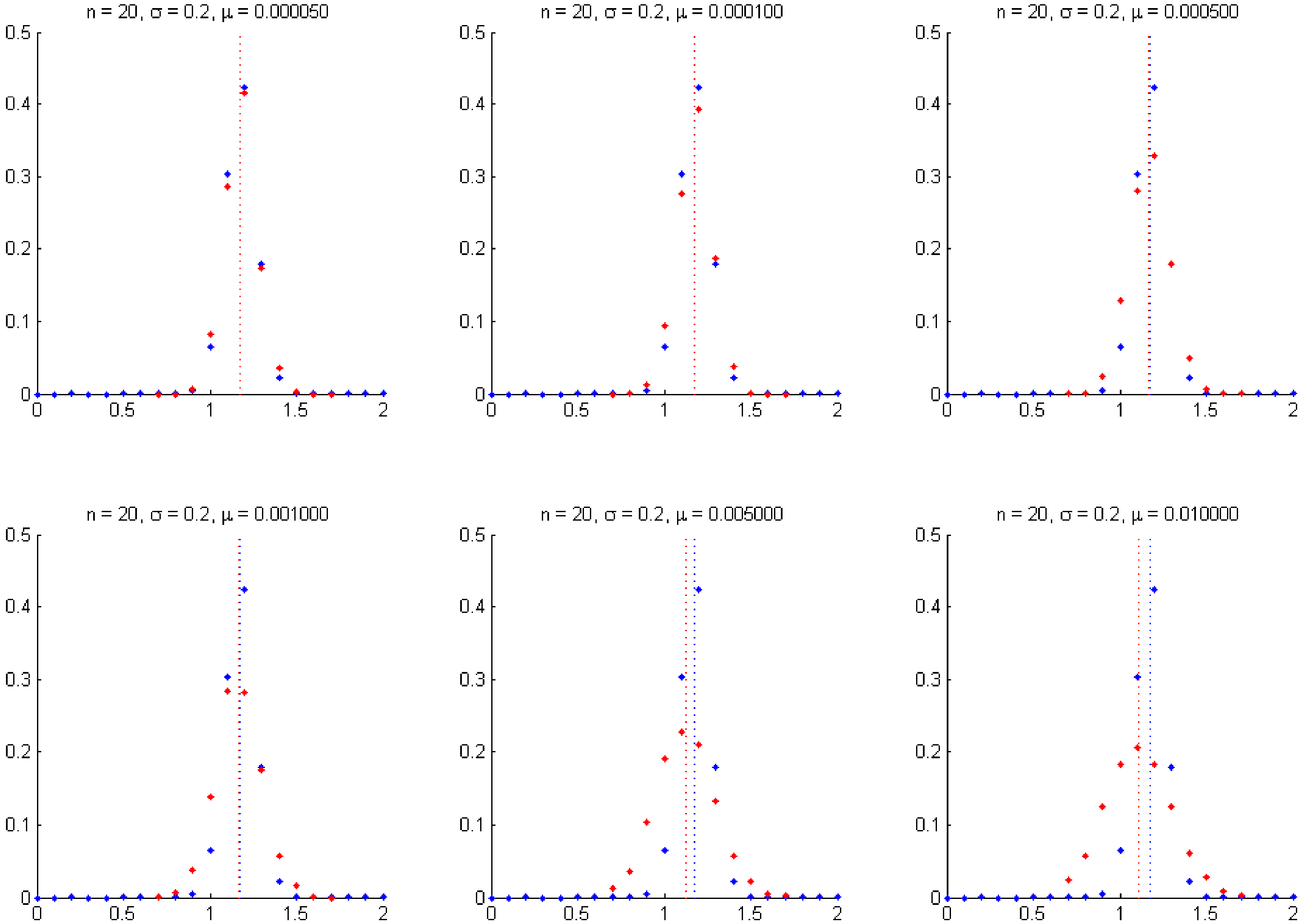
Analysis: $\mu \to 0$ (1)

- Economists typically analyze evolutionary models under the assumption that $\mu \to 0$ (i.e., almost no experimentation)
- From a mathematical point of view this is a convenient assumption
- For $\mu \to 0$ and $\sigma = 0$ (i.e., no noise), behavior in our model more or less coincides with behavior in the model of Eshel et al (*Amer. Econ. Rev.*, 1998)
- We use Markov chain theory to analyze our model for $\mu \to 0$ and $\sigma > 0$
- Unfortunately, due to the presence of normally distributed random variables in our model, no closed-form solutions can be obtained

Analysis: $\mu \rightarrow 0$ (2)



Analysis: Effect of μ (simulation)



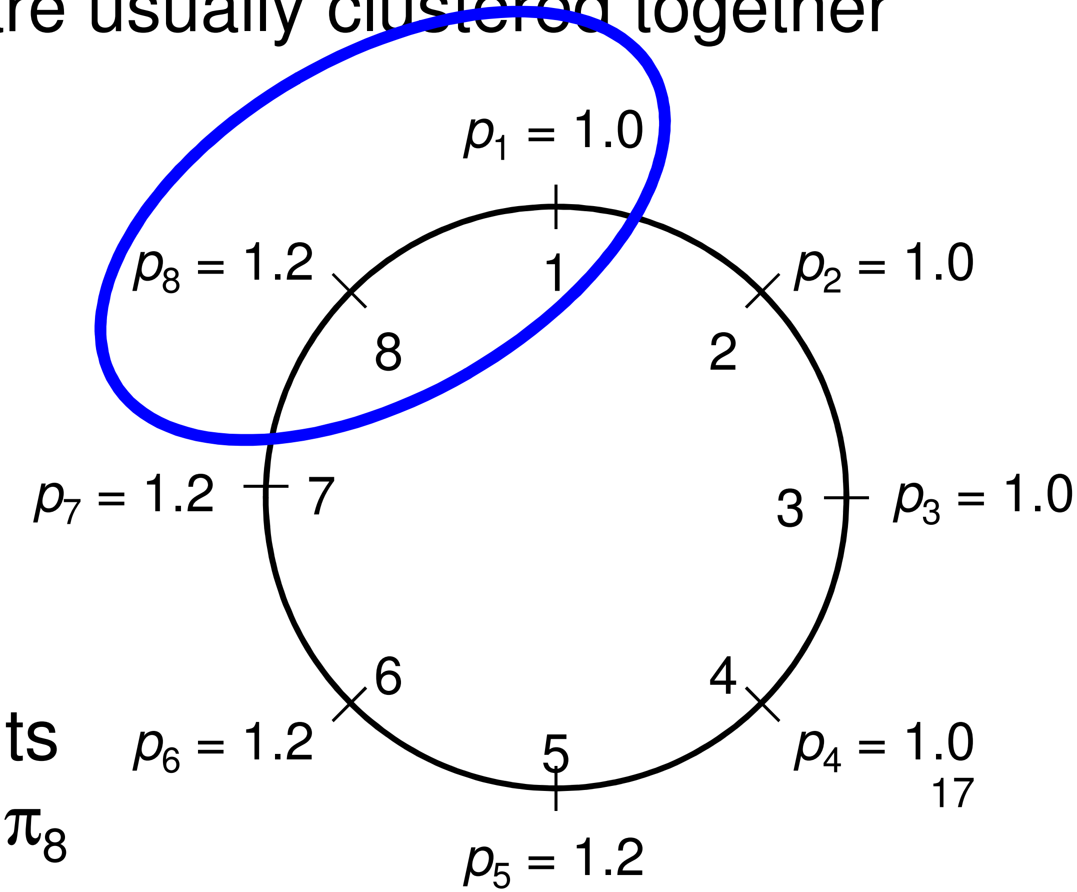
Intuition (1)

- Suppose firms charge a price of either 1.0 (i.e., the Nash equilibrium price) or 1.2
- Because firms tend to imitate their neighbors, firms charging the same price are usually clustered together
- Firms profits equal:

  $\pi_1 = 1.10$ $\pi_2 = 1.00$ $\pi_3 = 1.00$

  $\pi_4 = 1.10$ $\pi_5 = 1.08$ $\pi_6 = 1.20$

  $\pi_7 = 1.20$ $\pi_8 = 1.08$



- Firm 8 probably stays at its current price, since $\pi_1 < (\pi_7 + \pi_8) / 2$
- Firm 1 probably changes its price, since $(\pi_1 + \pi_2) / 2 < \pi_8$

Intuition (2)

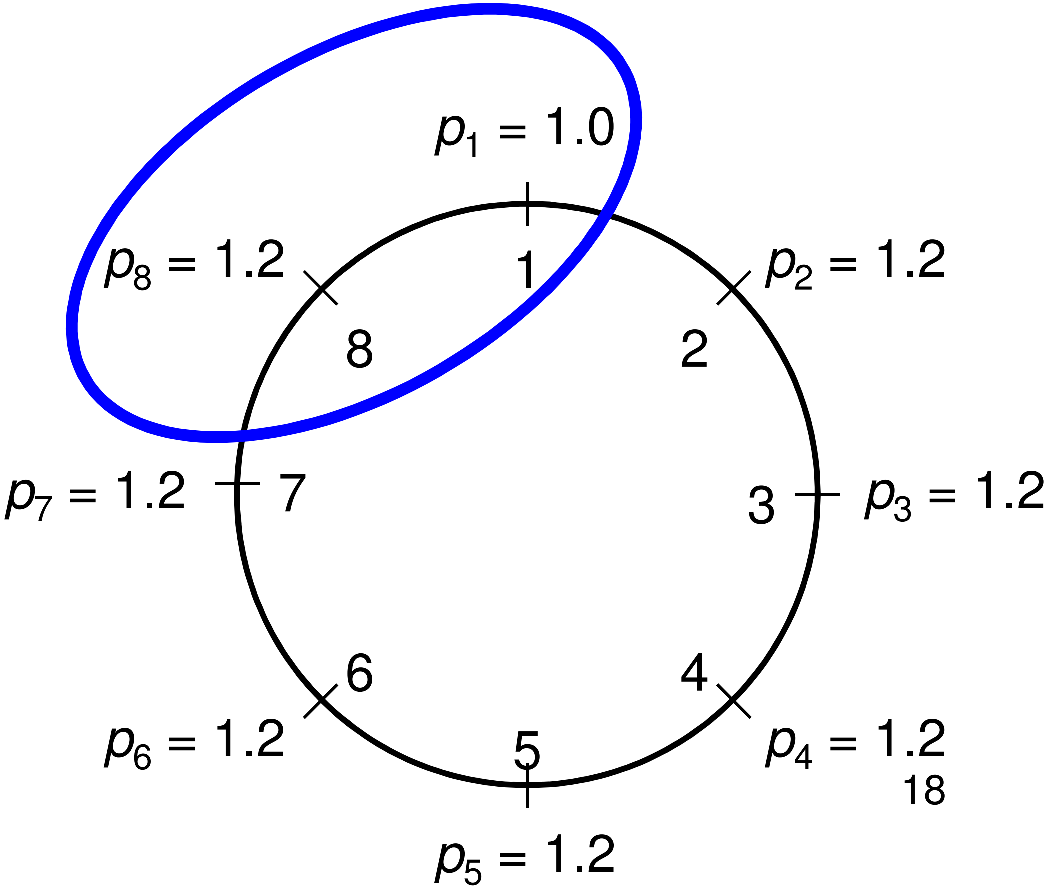
- However, when there are only a few lower-price firms left, these firms can easily take advantage of their higher-price neighbors
- Firms profits equal:

  $\pi_1 = 1.20$ $\pi_2 = 1.08$ $\pi_3 = 1.20$

  $\pi_4 = 1.20$ $\pi_5 = 1.20$ $\pi_6 = 1.20$

  $\pi_7 = 1.20$ $\pi_8 = 1.08$

- Firm 1 probably stays at its current price, since $\pi_1 > (\pi_2 + \pi_8) / 2$
- Firm 8 probably changes its price, since $\pi_1 > (\pi_7 + \pi_8) / 2$



Conclusions

- An evolutionary model with local interaction can result in cooperative behavior among spatially distributed price competing firms
- Hence, cooperative behavior among firms can be achieved without relying on the idea of reciprocity
- The amount of cooperation can be quite significant, with average profits up to 30% higher than in the Nash equilibrium
- Interestingly, the larger the number of firms, the more cooperative their behavior
- Our results remain largely unchanged when a significant amount of noise is introduced

Ongoing research

- We are currently studying extensions of our model in the following directions:
  - Firms imitating each other simultaneously rather than one at a time
  - Firms having global rather than local information about the profits of their competitors
  - Firms being located on a square lattice rather than on the circumference of a circle

Thank you for your attention!